

Hydrodynamical description of a hadron-quark first-order phase transition.

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Solutions of hydrodynamical equations are presented for the equation of state of the Van der Waals type allowing for the first order phase transition. Attention is focused on description of the hadron-quark phase transition in heavy ion collisions. It is shown that fluctuations dissolve and grow as if the fluid is effectively very viscous. Even in spinodal region germs are growing slowly due to viscosity and critical slowing down. This prevents enhancement of fluctuations in the near-critical region, which is frequently considered as a signal of the critical point in heavy ion collisions.

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There are many phenomena, where first-order phase transitions occur between phases with different densities. Description of such phenomena should be similar to that for the gas-liquid phase transition. Thereby it is worthwhile to find corresponding solutions of hydrodynamical equations. Though some simplified analytical [1, 2] and fragmentary two-dimensional numerical solutions [3] have been found, many problems remain unsolved. In nuclear physics different first-order phase transitions (e.g., to pion, kaon condensates and to the quark state) may occur in neutron stars [2, 4] and in heavy ion collisions [5, 6]. At low energies gas-liquid transition occurs [5]. It is also expected that at finite baryon density the hadron – quark gluon plasma (QGP) phase transition, which might manifest itself in violent nucleus-nucleus collisions, is of the first-order [6]. The hydrodynamical approach is efficient for description of heavy-ion collisions in a broad energy range (e.g. see [6, 7, 8]).

In this letter the dynamics of a first-order phase transition is described by equations of non-ideal non-relativistic hydrodynamics: the Navier-Stokes equation, the continuity equation, and general equation for the heat transport. We solve these equations numerically in two spatial dimensions, $d = 2$, and analytically for arbitrary d in the vicinity of the critical point. Then we perform estimations for the case of the hadron – QGP transition.

The best known example to illustrate principal features of a first order phase transition is the Van der Waals fluid. The pressure is given by $P_{\text{VW}}[n, T] = nT/(1 - bn) - n^2a$, where T is the temperature, n is the density of a conserving charge (e.g., the baryon charge), parameter a governs the strength of a mean field attraction and b controls a short-range repulsion. In practice we use a modified Van der Waals (mVW) equation of state (EoS): $P[n, T] = f(T)P_{\text{VW}}[n, T]$, where the function $f(T)$ is chosen so that $\frac{\partial P}{\partial T}|_{(n_{cr}, T_{cr})} = 0$ at the critical density n_{cr} and the critical temperature T_{cr} . We use $f(T) \simeq C(\delta T)[1 - 2(\delta T)]^2$, $\delta T = (T - T_{cr})/T_{cr}$, and the pre-factor $C(\delta T) = [1 + 4(\delta T)^2]^{-1}$ is chosen to reproduce ideal gas EoS for sufficiently low n and high T . This modification allows us to parameterize the EoS

with two minima in the free energy, being convenient for analytical treatment of the problem near the critical point (n_{cr}, T_{cr}) . We expand the quantities entering EoS and equations of hydrodynamics near a reference point (ρ_r, T_r) chosen somewhere in the vicinity of the critical point on the plane $P(\rho, T)$, where $\rho = mn$ is the mass density, m is the mass of the constituent. Assuming smallness of the velocity $\vec{u}(\vec{r}, \tau)$ of the germ we linearize hydrodynamical equations in u , density $\delta\rho = \rho - \rho_r$ and temperature $\delta T = T - T_r$. Applying then operator "div" to the Navier-Stokes equation and taking $z = \text{div } \vec{u}$ from the continuity equation we obtain [1, 2]:

$$\frac{\partial^2 \delta\rho}{\partial t^2} = \Delta \left[\delta P + \rho_r^{-1} (\tilde{d}\eta_r + \zeta_r) \frac{\partial \delta\rho}{\partial t} \right], \quad (1)$$

$\tilde{d} = 2(d-1)/d$. The pressure $\delta P = P - P[\rho_r, T_r] = \rho_r \frac{\delta[F(\delta\rho, T)]}{\delta(\delta\rho)}|_T$ is expressed through the Helmholtz free energy F for slightly inhomogeneous configurations; η_r and ζ_r are the first (shear) and second (bulk) viscosities; $\Delta = \partial_{x_1}^2 + \dots + \partial_{x_d}^2$.

Note that *thus derived Eq. (1) differs from the phenomenological Landau equation* for the nonconserving order parameter $\partial_t \phi = -\gamma(\delta F/\delta\phi)$, $\gamma = \text{const}$, and from equations used for the description of the dynamics of first-order phase transitions in heavy ion collisions [9] and in relativistic astrophysical problems [10]. The difference with the Landau equation disappears, if one sets zero the square bracketed term in the r.h.s. of Eq. (1). From the first glance, such a procedure is legitimate, if space-time gradients are small. However for a germ, being prepared in a fluctuation at $t = 0$ with a distribution $\delta\rho(t = 0, \vec{r}) = \delta\rho(0, \vec{r})$, the condition $\partial\delta\rho(t, \vec{r})/\partial t|_{t=0} \simeq 0$ should also be fulfilled (otherwise there appears a kinetic energy term). Two initial conditions cannot be simultaneously fulfilled, if the equation contains time derivatives of the first-order only. Thus, *there exists an initial stage of the dynamics of phase transitions ($t \lesssim t_{\text{init}}$), which is not described by the standard Landau equation.*

For low velocities the heat transport is described by the heat conductivity equation $c_V \frac{\partial T}{\partial t} = \kappa \Delta T$, where κ is

the heat conductivity and c_V is the specific heat. Time scale of the temperature relaxation is $t_T = R^2(t_T)c_V/\kappa$, where $R(t)$ is the size of the germ. On the other hand, time scale of the density relaxation, following Eq. (1), is $t_\rho \propto R$ (we show below that a germ of rather large size grows with constant velocity). Evolution of the germ is governed by the slowest mode. When sizes of germs begin to exceed the value R_{fog} , where R_{fog} is the size at which $t_T = t_\rho$, the growth is slowed down. Thus number of germs with the size $R \sim R_{\text{fog}}$ grows with time and *there appears a metastable state called the fog*.

For further convenience we choose $\rho_r = \rho_{cr}$, $T_r = T_{cr}$ and expand the Helmholtz free energy in $\delta\rho$ and δT :

$$\delta F = \int \frac{d^3x}{\rho_r} \left[\frac{c[\nabla(\delta\rho)]^2}{2} + \frac{\lambda(\delta\rho)^4}{4} - \frac{\lambda v^2(\delta\rho)^2}{2} - \epsilon\delta\rho \right], \quad (2)$$

$\delta F = F[\rho, T] - F[\rho_r, T_r]$. Then $a = \frac{9}{8} \frac{T_{cr}}{n_{cr}}$, $b = \frac{1}{3n_{cr}}$, $v^2 = -\frac{3m^2 T_{cr} \delta T}{2ab} = 4|\delta T| n_{cr}^2 m^2$, $\lambda = \frac{3ab}{2m^3} = \frac{9}{16} \frac{T_{cr}}{n_{cr}^2 m^3}$, $\epsilon = \frac{27(T_{cr} \delta T)^2}{4a} = 6n_{cr} T_{cr} (\delta T)^2$. Introducing dimensionless variables $\delta\rho = v\psi$, $\xi_i = x_i/l$, $i = 1, \dots, d$, $\tau = t/t_0$, we arrive at equation

$$-\beta \frac{\partial^2 \psi}{\partial \tau^2} = \Delta_\xi \left(\Delta_\xi \psi + 2\psi(1 - \psi^2) + \tilde{\epsilon} - \frac{\partial \psi}{\partial \tau} \right), \quad (3)$$

$$l = (2c/(\lambda v^2))^{1/2}, \quad t_0 = 2(\tilde{d}\eta_r + \zeta_r)/(\lambda v^2 \rho_r),$$

$$\tilde{\epsilon} = 2\epsilon/(\lambda v^3) = (8/3)|\delta T|^{1/2}, \quad \beta = c\rho_r^2/[\tilde{d}\eta_r + \zeta_r]^2.$$

Thus $l \propto |\delta T|^{-1/2}$ and $t_0 \propto |\delta T|^{-1}$.

There exists an opinion, cf. Ref. [11], that, if at some incident energy the trajectory passes in the vicinity of the critical point, the system may linger longer in this region due to divergence of susceptibilities that may reflect on observables. Contrary, we argue that *fluctuational effects in the vicinity of the critical point in heavy ion collisions can hardly be pronounced*, since all relevant processes are proved to be frozen for $\delta T \rightarrow 0$, while the system passes this region during a finite time.

To describe configurations of different symmetry we search two-phase solution of Eq. (3) in the form [1, 2],

$$\psi = \mp \tanh[\xi - \xi_0(\tau)] + \tilde{\epsilon}/4, \quad (4)$$

$\xi = \sqrt{\xi_1^2 + \xi_2^2 + \xi_3^2}$ for droplets/bubbles ($d_{\text{sol}} = 3$), $\xi = \sqrt{\xi_1^2 + \xi_2^2}$ for rods ($d_{\text{sol}} = 2$) and $\xi = \xi_1 = x/l$ for kinks ($d_{\text{sol}} = 1$) in $d = 3$ space. For $\tilde{\epsilon} > 0$ upper sign solution describes evolution of droplets (or rods and kinks of liquid phase) in a metastable super-cooled vapor medium. The lower sign solution circumscribes then bubbles (or kinks and rods of gas phase) in a stable liquid medium.

The boundary layer has the length $|\xi - \xi_0(\tau)| \sim 1$. Outside this layer corrections to homogeneous solutions are exponentially small. Considering motion of the boundary for $\xi_0(\tau) \gg 1$ we may put $\xi \simeq \xi_0(\tau)$ in (4). Then keeping only linear terms in $\tilde{\epsilon}$ in Eq. (3), we arrive at equation for $\xi_0(\tau)$:

$$\frac{\beta}{2} \frac{d^2 \xi_0}{d\tau^2} = \pm \frac{3}{2} \tilde{\epsilon} - \frac{d_{\text{sol}} - 1}{\xi_0(\tau)} - \frac{d\xi_0}{d\tau}. \quad (5)$$

Substituting (4) in (2) we obtain

$$\delta F[\xi_0] = \frac{2\pi^{3/2} \Lambda^{3-d_{\text{sol}}} \lambda v^4 l^{d_{\text{sol}}}}{\Gamma(d_{\text{sol}}/2) \Gamma(1 + (3 - d_{\text{sol}})/2) \rho_r} \quad (6)$$

$$\times \left[\mp \tilde{\epsilon} \xi_0^{d_{\text{sol}}}/d_{\text{sol}} + 2\xi_0^{d_{\text{sol}}-1}/3 \right],$$

2Λ is the diameter, height of cylinder and the length of the squared plate for $d_{\text{sol}} = 3, 2$ and 1 , respectively; Γ is the Euler Γ -function. The first term in (6) is the volume term and the second one is the surface contribution, δF_{surf} . At fixed volume in $d = 3$ space, the surface contribution for droplets/bubbles is smaller than for rods and slabs. Thereby if a germ prepared in a fluctuation is initially nonspherical it acquires spherical form with passage of time. Surface term is $\delta F_{\text{surf}} \equiv \sigma S$, S is the surface of the germ, σ is the surface tension, and the gradient term in (2) is then $\delta F_{\text{surf}}^{\text{grad}} = \frac{2v^2 c}{3l\rho_r} S = \frac{1}{2} \delta F_{\text{surf}}$. Thus we are able to find relations: $\sigma = \sigma_0 |\delta T|^{3/2}$, $\sigma_0^2 = 32mn_{cr}^2 T_{cr} c$; $l = \frac{\sigma_0}{6T_{cr} n_{cr} |\delta T|^{1/2}}$. There are two dimensionless parameters in (3) and (5): $\tilde{\epsilon}$ and β . The value $\tilde{\epsilon}$ distinguishes metastable and stable state minima in the free energy, $\beta = (32T_{cr})^{-1} [\tilde{d}\eta_r + \zeta_r]^{-2} \sigma_0^2 m$ controls dynamics. *The larger viscosity and the smaller surface tension, the effectively more viscous is the fluidity of germs*. For $\beta \ll 1$ one deals with effectively viscous fluid and at $\beta \gg 1$, with perfect fluid.

At hand of Eq. (5) consider analytically several typical solutions for germ evolution. Consider evolution of germs of stable phase in metastable matter.

1) *Short time evolution of a germ*. For small τ (initial stage) using Taylor expansion in τ and assuming zero initial velocity, $\frac{d\xi_0}{d\tau}|_{\tau=0} \simeq 0$, we obtain

$$R(t) \simeq R_0 + (wt^2/2) [1 - 2t/(3t_0\beta)]$$

valid for $t \ll (\frac{R_0}{w})^{1/2}$ and $t \ll t_{\text{init}} = \frac{2(\tilde{d}\eta_r + \zeta_r)\beta}{\lambda v^2 m n_{cr}} \propto \frac{\sigma_0^2}{(\tilde{d}\eta_r + \zeta_r)|\delta T|}$. Initial stage of the process proceeds with acceleration

$$w = (d_{\text{sol}} - 1) \lambda v^2 (R_0 - R_{cr}) / (R_0 R_{cr}),$$

which changes sign at the initial size $R_0 = R_{cr}$ where

$$R_{cr} = (d_{\text{sol}} - 1) v^2 \sqrt{2c\lambda} / (3|\epsilon|) \propto 1/|\delta T|$$

is the critical size. Germs with $R_0 < R_{cr}$ shrink, while germs with $R_0 > R_{cr}$ grow. For germs with $|R_0 - R_{cr}| \ll R_{cr}$ the size changes very slowly ($w \propto |\delta T|(R_0 - R_{cr})/R_{cr}^2$). For undercritical germs of a small size, $w \propto -|\delta T|/R_0$. Slabs of stable phase, being placed in a metastable medium, grow independently of what was their initial size. Note that the same value R_{cr} follows from minimization of the free energy (6).

2) *Long time evolution of a large germ*. For $t \gg t_{\text{init}}$, we may drop the term $\partial^2 \xi_0 / \partial \tau^2$ in the l.h.s of Eq. (5). For $R(t) \gg R_{cr}$, surface effects become unimportant and we arrive at the solution

$$R(t) \simeq R_0 + u_{\text{asympt}} t, \quad u_{\text{asympt}} = 3|\epsilon| \sqrt{\beta} / \sqrt{2\lambda v^4}.$$

Germ grows with constant velocity. The time scale for the growth of the germ with size $R \gg R_{cr}$ is $t_\rho = R/u_{\text{asympt}} = (m/T_{cr})^{1/2} R / ((18\beta)^{1/2} |\delta T|)$. Asymptotic regime is reached at very large values of time, provided the system is near the critical point.

3) *Long time evolution of a small germ.* Describing germs of a small size ($l \ll R \ll R_{cr}$, $d_{\text{sol}} \neq 1$) for $t \gg t_{\text{init}}$, we can drop the term $\propto \tilde{\epsilon}$ in (5). Then solution acquires the form

$$R(t) \simeq \sqrt{R_0^2 - 2(d_{\text{sol}} - 1)tl^2/t_0}.$$

The time scale at which the initial germ of a small size dissolves is, $t_{\text{dis}} = \frac{16n_{cr}T_{cr}(\bar{d}\eta_s + \zeta_r)R_0^2}{(d_{\text{sol}} - 1)\sigma_0^2}$, and is $\propto R_0^2$. Thus, fluctuations of sufficiently small sizes are easily produced and dissolve rapidly.

4) *Fluctuations in spinodal region.* Let the system be driven to a spinodal region where fluctuations of even infinitesimally small amplitudes and sizes may grow into a new phase. To demonstrate this we take the free energy δF to be close to its maximum ($\delta F \simeq 0$). Then we linearize Eq. (3) dropping ψ^3 term. Setting $\psi = -\frac{\tilde{\epsilon}}{2} + \text{Re}\{\psi_0 e^{\gamma_\psi \tau + i\vec{k}\vec{\xi}}\}$, ψ_0 is an arbitrary but small real constant, we find two solutions,

$$\gamma_\psi(k) = (-k^2 \pm \sqrt{k^4 + 8\beta k^2 - 4\beta k^4}) / (2\beta). \quad (7)$$

Growing modes correspond to the choice of "+"-sign and $k^2 < 2$. The time scale at which an aerosol of germs develops is $t_{\text{aer}} = t_0/\gamma_\psi(k_m)$, k_m corresponds to $\max\{\gamma_\psi(k)\}$. For an effectively large viscosity ($\beta \ll 1$) there are two solutions: the damped one, and the growing one for $k < \sqrt{2}$. The most rapidly growing mode is $\gamma_\psi(k_m) \simeq 2$, $k_m = 2\beta^{1/4} \ll 1$. The time scale characterizing growth of this mode is $t_{\text{aer}}^\eta \sim \frac{1}{2}t_0 = \frac{4(\bar{d}\eta_0 + \zeta_0)}{9n_{cr}T_{cr}|\delta T|}$. The typical size of germs, $R_{\text{aer}}^\eta \simeq l/(2\beta^{1/4})$, increases with an increase of the viscosity. For $k^2 > 2$ both modes are damped. In the case of an effectively small viscosity ($\beta \gg 1$) we get $\gamma_\psi(k) \simeq \pm k\sqrt{2/\beta}\sqrt{1 - k^2/2}$, and $\gamma_\psi^{\text{max}}(k_m = 1) = \beta^{-1/2}$. The time scale characterizing growing modes, $t_{\text{aer}}^{\text{id}} \sim t_0/\gamma_\psi = 2c^{1/2}/(\lambda v^2) \propto \delta T^{-1}$, does not depend on the viscosity in this limit. The size scale of germs is $R_{\text{aer}}^{\text{id}} \simeq l$. Modes with $k^2 > 2$ oscillate and do not grow into a stable phase.

For the description of the hadron-QGP first-order phase transition we take values $T_{cr} \simeq 162$ MeV, $n_{cr}/n_{\text{sat}} \simeq 1.3$, as they follow from lattice calculations, see [12]. Parameters of the EoS are then as follows: $a \simeq 8.76 \cdot 10^2 (\text{MeV} \cdot \text{fm}^3)$, $b \simeq 1.60 \text{ fm}^{-3}$, $\lambda \simeq 7.80 \cdot 10^{-5} q^{-3} (\text{fm}^6/\text{MeV}^2)$, $v^2 \simeq 1.56 \cdot 10^4 q^2 |\delta T| (\text{MeV}^2/\text{fm}^6)$, $\epsilon \simeq 2.02 \cdot 10^2 (\delta T)^2 (\text{MeV}/\text{fm}^3)$, where m is the effective quark mass, $q = (m/300 \text{ MeV})$. Further we obtain $l(T = 0) \simeq 0.2$ fm (radius of confinement) for $\sigma_0 \simeq 40 \text{ MeV}/\text{fm}^2$. If one used $\sigma_0 \simeq 100 \text{ MeV}/\text{fm}^2$, one would estimate $l(T = 0) \simeq 0.5$ fm.

Next we use $s \simeq 7T^3/(T/T_{cr})$ at T near T_{cr} , $c_V \simeq 28T^3/(T/T_{cr})$, as it follows from the lattice data [12]. Assuming the minimal value of the viscosity $\eta_{\text{min}} = s/4\pi \simeq$

$60 \text{ MeV}/\text{fm}^2$, $\zeta_{\text{min}} = 0$ we evaluate $\beta_{\text{QGP}}^{\text{max}} \simeq 0.015q$ for $\sigma_0 \simeq 40 \text{ MeV}/\text{fm}^2$, that corresponds to the limit of *effectively very large viscosity*. Even for $\sigma_0 \simeq 100 \text{ MeV}/\text{fm}^2$, $m = 600 \text{ MeV}$ we would get $\beta_{\text{QGP}}^{\text{max}} \simeq 0.2 \ll 1$. Note that following [13] the bulk viscosity diverges in the critical point. If were so ($\beta \rightarrow 0$), the quark-hadron system would behave as absolutely viscous fluid, like glass, in near critical region. Contrary, Ref. [14] argues for a smooth behavior of the bulk viscosity.

With $\beta = 0.015$ we further estimate $t_0 \simeq 2|\delta T|^{-1} \text{ fm}$, $t_\rho \simeq 2.6Rq^{1/2}|\delta T|^{-1}$, and $t_{\text{dis}} \simeq 14qR_0(R_0/\text{fm})$. Typical time for the formation of the aerosol is $t_{\text{aer}}^\eta \simeq |\delta T|^{-1} \text{ fm}$, and typical size of germs in aerosol is $R_{\text{aer}}^\eta \simeq 0.24|\delta T|^{-1/2} \text{ fm}$. Only $t_{\text{init}} \simeq 0.03q|\delta T|^{-1} \text{ fm}$ proves to be small (excluding quite small δT). Critical slowing down that limits growing of the σ meson correlation length was discussed in [15].

For the thermal conductivity we use an estimation $\kappa_{\text{QGP}} \simeq \alpha_0\eta/m$ taking $\alpha_0 \simeq 3$ to recover the relation between values of κ and η for nuclear gas-liquid phase transition at low energies [16]. The scale of the heat transport time is $t_T \simeq 26q(R/\text{fm})^2 \text{ fm}$. Using that $R_{cr} \simeq 0.1|\delta T|^{-1} \text{ fm}$, we obtain $R_{\text{fog}} \simeq 0.1q^{-1/2}|\delta T|^{-1} \text{ fm} \lesssim R_{cr}$. The value R_{fog} proved to be very small ($\lesssim 0.1 \div 1 \text{ fm}$). However, time scale t_T is rather long. Therefore, the system most probably would have no time to fully develop a fog-like state in a hadron-quark phase transition in heavy ion collisions.

For the system in the vicinity of the critical point all estimated time scales (except t_{init}) are very large. If the system trajectory paths rather far from the critical point (T_{cr}, ρ_{cr}), all time scales, except t_T , become less than the typical life-time of the fireball ($\sim 10 \text{ fm}$ at RHIC conditions). Reynolds numbers are $\lesssim 1$, being much smaller than the critical value ($\gtrsim 1000$). Thereby, turbulence regime is not reached.

We solved numerically the general system of equations of nonideal hydrodynamics for $d = 2$. To illustrate the results we consider dynamics of overcritical and undercritical germs (disks) in infinite matter taking initial density profile as $\rho(x, y; t = 0) = \rho_{\text{out}} + (\rho_{\text{in}} - \rho_{\text{out}})\Theta(R_0 - r)$, $r = \sqrt{x^2 + y^2}$, ρ_{in} and ρ_{out} are densities in stable and metastable homogeneous phases, respectively.

In Fig. 1 we show the time evolution of a liquid disk (upper panel) and a gas disk (lower panel) for $T/T_{cr} = 0.85$. In the middle column we show dynamics of an initially overcritical germ with $R_0 = 0.3L > R_{cr} \simeq 0.16L$ and in the right column, of undercritical germ $R_0 = 0.1L$, $L = 5 \text{ fm}$. The time snapshots are shown in Figure in units L . The configuration is computed for values of kinetic parameters $\eta \simeq 45 \text{ MeV}/\text{fm}^2$ and $\beta \simeq 0.2$. We see that in case $R_0 > R_{cr}$ (middle column) disks slowly grow with the time passage. For overcritical discs the initially selected distribution acquires the tanh-like shape, see (4), only for $t \gtrsim (50 \div 100)L$. Initial disks of a small size practically disappear for $t \gtrsim 10L = 50 \text{ fm}$. Due to the matter admission to the disk surface and the shape

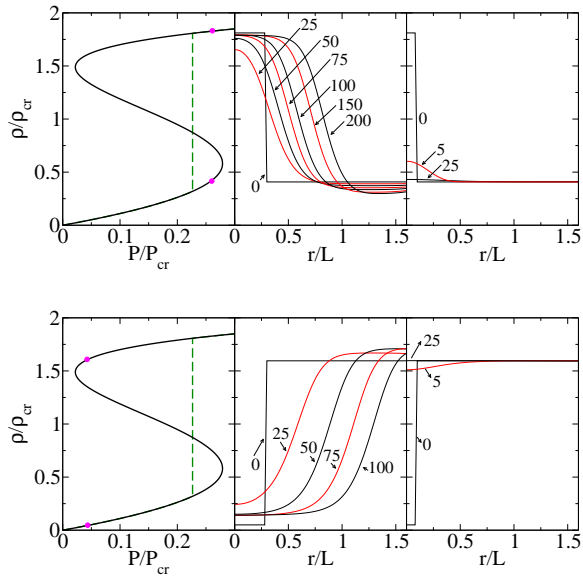


FIG. 1: Isotherm for the pressure as function of the density with initial and final states shown by dots (left column). Dash vertical line shows the Maxwell construction, MC. In the upper panel the initial state corresponds to the stable liquid phase disk in the metastable super-cooled gas, and in the lower panel, to a stable gas phase disk in the metastable super-heated liquid. Middle column shows time evolution of density profiles for the overcritical liquid disk (upper panel) and gas disk (lower panel). Numbers near curves (in L) are time snapshots. Right column, the same for initially undercritical liquid or gas disk.

reconstruction, the density decreases in the liquid disk neighborhood below the value of the density in the homogeneous metastable matter and it increases in the gas disk surrounding above the value of the density in the homogeneous metastable matter (see the middle column).

In Fig. 2 we demonstrate time evolution of the wave amplitudes, $\rho(t) = \bar{\rho} + A_0 f(t) \sin(\vec{k}\vec{r})$, for an undercritical value of the wave number k (left panel) and for an overcritical value (right panel). In case of the overcritical k

and effectively small viscosity ($\beta = 20$) we demonstrate change of the amplitude in the $3/2$ -periods of the oscillation. Such a behavior fully agrees with that follows from our analytical treatment of the problem.

Concluding, even in the spinodal region germs are growing slowly, if the system is somewhere in the vicinity of the critical point. Thus in heavy ion collisions the expanding fireball may linger in the QGP state, until $T(t)$ decreases below the corresponding equilibrium value of the temperature of the phase transition. There exists a belief that strongly coupled QGP state, represents almost perfect fluid [8]. We demonstrate *the essential role of viscosity and surface tension in dynamics of first-order phase transitions, including the hadron-QGP one*. Fluctuations in QGP (at a finite baryon density) grow and dissolve as if the fluid were very viscous. Variation of parameters in broad limits does not change conclusions.

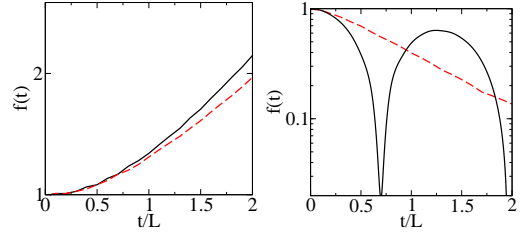


FIG. 2: Time evolution of wave amplitudes $f(t)$ in aerosol for effectively small ($\beta = 20$, solid line) and large ($\beta = 0.2$, dash line) viscosity. Left panel: $k = 2l/L$ (growing modes). Right: $k = 8l/L$ (oscillation modes for large β and damping modes for small β). Other parameters are the same as in Fig 1.

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